cosine are measured along the X-axis, while the sine and the cosecant are measured along the Y-axis. The following facts are evident from an inspection of the figure for an angle in each quadrant:

The algebraic signs of the sine, cosine, secant, and cosecant are determined by the direction in which each is measured from O in accordance with the usual convention of Analytics. The algebraic sign of the tangent is plus when the tangent is measured to the right of OT (as one looks from O); minus, when measured to the left. The tangent and cotangent have always the same sign.

Any two functions of an angle measured along the same line have unity for their product.

It seems to me that this representation of the functions will make it easier for the student to fix the algebraic sign of any function in any quadrant, and also to remember the group of products each equal to unity.

The method also lends itself very readily to approximate measurements of the functions for rough work. For this purpose the pupil will require a circular protractor and a "square" graduated to tenths of the unit on the inner edges of the angle. This square should have for unit the radius of the protractor. Tangents and cotangents may be read (accurately to tenths, estimated to hundredths) by laying one inner edge of the square along the radius OT which cuts off on the protractor the required angle, and then reading TM or TN according as tangent or cotangent is desired. The sine and the cosine may be read by putting the vertex of the square as at P and reading PO and PT. (It is in measuring tangents and cotangents, of course, that this method has the advantage over the ordinary methods.)

NOTE ON AN APPROXIMATION IN TRIGONOMETRY.

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The question as to whether it is possible to find the angles of a triangle in terms of the sides approximately correct, without the use of tables, is one which is often asked by persons who do not know how to use tables and yet find it necessary to use the approximate values of the angles. They understand mensuration and naturally wonder why there is not a formula for this purpose given.

The following simple deductions lead readily to such a formula.

Let A be the smallest angle of a triangle. Let the sides be denoted by a, b, c, and the area by $\triangle$. Also let $2s=a+b+c$.

Then, $\sin A=\dfrac{2\triangle}{bc}$, $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$, $2+\cos A=\dfrac{4bc+b^2+c^2-a^2}{2bc}$,

$2+\cos A=\frac{2s\,(s-a)+bc}{bc}$. Hence,

$$\frac{2\triangle}{2s\,(s-a)+bc}=\frac{\sin A}{2+\cos A}=\frac{A-\frac{1}{6}A^3}{3-\frac{1}{2}A^2},$$

omitting higher powers, or $\frac{2\triangle}{2s\,(s-a)+bc}=\frac{A}{3}$, in linear measure, $=\frac{\pi A}{3\times 180}$ in circular measure.

That is, $A=\frac{1080\triangle}{\pi[2s(s-a)+bc]}=\frac{344\triangle}{2s\,(s-a)+bc}$.

As a special case, let $a=20$, $b=51$, $c=65$. Then $\triangle=408$, $s=68$, $s-a=48$, $bc=3315$. Hence, $A=\frac{140352}{9843}=14.259^\circ=14^\circ\,15'\,3''$.

Similarly, $B=\frac{344\triangle}{2s\,(s-b)+ac}=\frac{140352}{3612}=38.857^\circ=38^\circ\,51'\,25''$.

The values by the tables are: $A=14^\circ\,15'$, $B=38^\circ\,52'\,48''$.

If we have a right triangle, then $a^2+b^2=c^2$, and

$$A=\frac{172ab}{\frac{1}{2}\,(b^2+2bc+c^2-a^2)+bc}=\frac{172ab}{b^2+2bc}=\frac{172a}{b+2c}.$$

This value for the lesser angle of a right triangle, I am informed, is found in some work, but I cannot learn where or how it is deduced. I will greatly appreciate this information from some reader of the MONTHLY.

As an application of this formula, let $a=4476$, $b=7332.8$, $c=8590$. Then $A=\frac{769872}{24512.8}=31.407^\circ=31^\circ\,24'\,25''$. The value by the tables is $A=31^\circ\,24'$. The smaller the angle the less the error. Hence it is always best to find the small angle by this formula.